



MODEL ANSWERS

Please write clearly in block capitals.

Centre number

Candidate number

Surname _____

Forename(s) _____

Candidate signature _____

I declare this is my own work.

AS FURTHER MATHEMATICS

Paper 2 Statistics

Thursday 14 May 2020

Afternoon

Time allowed: 1 hour 30 minutes

Materials

- You must have the AQA formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a scientific calculator that meets the requirements of the specification. (You may use a graphical calculator.)
- You must ensure you have the other optional Question Paper/Answer Book for which you are entered (**either** Discrete **or** Mechanics). You will have 1 hour 30 minutes to complete **both** papers.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 40.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
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2	
3	
4	
5	
6	
7	
8	
TOTAL	



Answer **all** questions in the spaces provided.

- 1 The discrete random variable X has the following probability distribution function.

$$P(X = x) = \begin{cases} 0.2 & x = 1 \\ 0.3 & x = 2 \\ 0.1 & x = 3, 4 \\ 0.25 & x = 5 \\ 0.05 & x = 6 \\ 0 & \text{otherwise} \end{cases}$$

Find the mode of X .

Circle your answer.

[1 mark]

0.1

0.25

2

3

mode = most appearing i.e. has the highest probability.

highest probability = 0.3

\therefore mode = 2



- 2 A χ^2 test is carried out in a school to test for association between the class a student belongs to and the number of times they are late to school in a week.

The contingency table below gives the expected values for the test.

		Number of times late				
		0	1	2	3	4
Class	A	8.12	14	15.12	14	4.76
	B	8.99	15.5	16.74	15.5	5.27
	C	11.89	20.5	22.14	20.5	6.97

Find a possible value for the degrees of freedom for the test.

Circle your answer.

[1 mark]



8

12

15

$$\begin{aligned} \text{Degree of freedom} &= (\text{rows} - 1) (\text{columns} - 1) \\ &= (3 - 1) (4 - 1) \end{aligned}$$

Turn over for the next question

$$= 2 \times 3 = 6$$

Turn over ►



- 3 The random variable X represents the value on the upper face of an eight-sided dice after it has been rolled. The faces are numbered 1 to 8

The random variable X is modelled by a discrete uniform distribution with $n = 8$

- 3 (a) Find $E(X)$

[1 mark]

x	1	2	3	4	5	6	7	8
$P(X=x)$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

$$E(X) = \frac{1}{8} \times (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8) = 4.5$$

- 3 (b) Find $\text{Var}(X)$

[1 mark]

Using the table above,

x^2	1	4	9	16	25	36	49	64
$P(X^2=x^2)$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

$$E(X^2) = \frac{1}{8} (1 + 4 + 9 + 16 + 25 + 36 + 49 + 64) = 25.5 - (E(X))^2 = 25.5 - 4.5^2 = 5.25$$

- 3 (c) Find $P(X \geq 6)$

[1 mark]

$$P(X \geq 6)$$

The possible options are; 6, 7, 8

$$\therefore \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = 0.375$$



3 (d) The dice was rolled 800 times and the results below were obtained.

x	1	2	3	4	5	6	7	8
Frequency	103	63	84	110	74	41	85	240

State, with a reason, how you would refine the model for the random variable X .

[2 marks]

↳ The dice appears to be biased.
↳ The random variable X would be modelled with a discrete random variable where the probabilities are estimated using relative frequencies

Turn over for the next question

Turn over ►



4 Murni is investigating the annual salary of people from a particular town.

She takes a random sample of 200 people from the town and records their annual salary.

The mean annual salary is £28 500 and the standard deviation is £5100

Calculate a 97% confidence interval for the population mean of annual salaries for the people who live in the town, giving your values to the nearest pound.

[3 marks]

The z value for 97% confidence interval = 2.17

$$\therefore CI = \bar{x} \pm z \sqrt{\frac{s^2}{n}}$$

$$\therefore 28500 \pm 2.17 \sqrt{\frac{5100^2}{200}}$$

$$UL = 29283 \text{ (nearest pound)}$$

$$LL = 27717 \text{ (nearest pound)}$$

$$\Rightarrow (27717, 29283)$$



Turn over for the next question

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5 The discrete random variable X has the following probability distribution.

x	2	4	6	9
$P(X = x)$	0.2	0.6	0.1	0.1

5 (a) Find $P(X \leq 6)$

[1 mark]

$$P(X \leq 6) = 1 - P(X = 9)$$

$$= 1 - 0.1 = 0.9$$

5 (b) Let $Y = 3X + 2$

Show that $\text{Var}(Y) = 32.49$

[5 marks]

We know: $\text{Var}(3X + 2)$

$$= 3^2 \text{Var}(X)$$

$$\textcircled{1} = 9 \text{Var}(X)$$

Step 1
calculate:
 $E(X)$

$$= (2 \times 0.2) + (4 \times 0.6)$$

$$+ (6 \times 0.1) + (9 \times 0.1)$$

$$= 4.3$$

Step 2
calculate:
 $E(X^2)$

$$= (2^2 \times 0.2)$$

$$+ (4^2 \times 0.6)$$

$$+ (6^2 \times 0.1)$$

$$+ (9^2 \times 0.1)$$

$$= 22.1$$

Step 3

Using the formula
 $\text{Var}(X) = E(X^2) - (E(X))^2$

$$= 22.1 - 4.3^2$$

$$= 3.61$$

Step 4

$$9(3.61) = 32.49$$

as required



5 (c) The continuous random variable T is independent of Y .

Given that $\text{Var}(T) = 5$, find $\text{Var}(T + Y)$

[1 mark]

$$\text{Var}(T+Y) = \text{Var}(T) + \text{Var}(Y)$$

$$= 5 + 32.49 \text{ (from above)}$$

$$= 37.49$$

Turn over for the next question

Turn over ►



6 The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{4}{45}(x^3 - 10x^2 + 29x - 20) & 1 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

6 (a) Find $P(X < 2)$

[2 marks]

$$\frac{4}{45} \int_1^2 x^3 - 10x^2 + 29x - 20 dx$$

$$\Rightarrow \frac{4}{45} \left[\frac{x^4}{4} - \frac{10x^3}{3} + \frac{29x^2}{2} - 20x \right]_1^2$$

$$\Rightarrow \frac{4}{45} \left[\frac{2^4}{4} - \frac{10(2)^3}{3} + \frac{29(2)^2}{2} - 20(2) \right] - \left[\frac{1}{4} - \frac{10}{3} + \frac{29}{2} - 20 \right]$$

$$\Rightarrow \left[\frac{2^4}{4} - \frac{10(2)^3}{3} + \frac{29(2)^2}{2} - 20(2) \right] - \left[\frac{1}{4} - \frac{10}{3} + \frac{29}{2} - 20 \right] = \frac{-14}{3}$$

$$\frac{4}{45} \left[\frac{-14}{3} - \left(-\frac{103}{12} \right) \right] = \frac{47}{135}$$

6 (b) Verify that the median of X is 2.3, correct to two significant figures.

[4 marks]

Equate integral $\int_1^m f(x) = 0.5$

From above

$$\frac{4}{45} \left[\left[\frac{m^4}{4} - \frac{10(m)^3}{3} + \frac{29m^2}{2} - 20m \right] - \left[\frac{1}{4} - \frac{10}{3} + \frac{29}{2} - 20 \right] \right] = 0.5$$

$$\left[\frac{1}{4} - \frac{10}{3} + \frac{29}{2} - 20 \right] = 0.5$$

$$\frac{4}{45} \left[\frac{m^4}{4} - \frac{10m^3}{3} + \frac{29m^2}{2} - 20m + \frac{103}{12} \right] = 0.5$$

$$\Rightarrow \frac{m^4}{45} - \frac{8m^3}{27} + \frac{58m^2}{45} - \frac{16m}{9} + \frac{103}{135} = 0.5$$

$$\Rightarrow \frac{m^4}{45} - \frac{8m^3}{27} + \frac{58m^2}{45} - \frac{16m}{9} + \frac{71}{270} = 0$$

$$m = 2.282817 \dots \therefore m = 2.3 \text{ (2sf)}$$



6 (c) Find the mean of X .

[2 marks]

$$\int_a^b x f(x) \Rightarrow \int_1^4 \frac{4}{45} x (x^3 - 10x^2 + 29x - 20) dx$$

$$\Rightarrow \frac{4}{45} \int_1^4 (x^4 - 10x^3 + 29x^2 - 20x) dx$$

$$\Rightarrow \frac{4}{45} \left[\frac{x^5}{5} - \frac{10x^4}{4} + \frac{29x^3}{3} - \frac{20x^2}{2} \right]_1^4$$

$$\Rightarrow \frac{4}{45} \left[\left[\frac{4^5}{5} - \frac{10(4)^4}{4} + \frac{29(4)^3}{3} - 10(4)^2 \right] - \left[\frac{1}{5} - \frac{10}{4} + \frac{29}{3} - 10 \right] \right]$$

Turn over for the next question

$$\left[\frac{1}{5} - \frac{10}{4} + \frac{29}{3} - 10 \right]$$

$$\Rightarrow \frac{4}{45} \left[\frac{352}{15} - - \frac{79}{3} \right]$$

$$= \frac{58}{25} = 2.32$$

Turn over ►



7 A restaurant has asked Sylvia to conduct a χ^2 test for association between meal ordered and age of customer.

7 (a) State the hypotheses that Sylvia should use for her test.

[1 mark]

H_0 : There is no association between the meal ordered and the age of the customer.

H_1 : There is an association between the meal ordered and age of customer

7 (b) Sylvia correctly calculates her value of the test statistic to be 44.1

She uses a 5% level of significance and the degrees of freedom for the test is 30.

Sylvia accepts the null hypothesis.

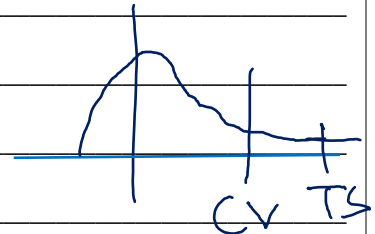
Explain whether or not Sylvia was correct to accept the null hypothesis.

[4 marks]

χ^2 critical value for $DOF = 30$ is
43.773 @ 5% significance level

$$43.773 < 44.1$$

\therefore Reject H_0



\therefore Sylvia was **not correct** to accept H_0



7 (c) State in context the correct conclusion to Sylvia's test.

[1 mark]

-> There is significant evidence to suggest that there is an association between the meal ordered and gender of customer.

Turn over for the next question

Turn over ►



8 There are two hospitals in a city.

Over a period of time, the first hospital recorded an average of 20 births a day.

Over the same period of time, the second hospital recorded an average of 5 births a day.

Stuart claims that birth rates in the hospitals have changed over time.

On a randomly chosen day, he records a total of 16 births from the two hospitals.

8 (a) Investigate Stuart's claim, using a suitable test at the 5% level of significance.

[6 marks]

$$H_0: \lambda = 25 \quad (20 + 5 = \text{Total Births})$$

$$H_1: \lambda \neq 25.$$

$X \sim P_0(25)$ and x is the no. of Births.

$$p(X \leq 16) = 0.038$$

Since it is a 2-T-T we use $\frac{0.05}{2} = 0.025$

$$0.038 > 0.025$$

\therefore Accept H_0

\therefore There is no significant evidence to suggest that the total Birth rate in the 2 hospitals has changed

2TT \rightarrow



- 8 (b) For a test of the type carried out in part (a), find the probability of making a Type I error, giving your answer to two significant figures.

[3 marks]

$$P(X \leq 15) = 0.02229$$

The other part of CR

$$P(X > 36) = 0.02246$$

$$\begin{aligned} \therefore P(\text{Type I error}) &= 0.02229 + \\ &\quad 0.02246 \\ &= 0.045 \end{aligned}$$

END OF QUESTIONS



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